
A one-dimensional finite element simulation for the fire-performance of GRP panels for offshore structures

Model for GRP
fire-performance

609

M.R.E. Looyeh

Department of Marine Technology, University of Newcastle-upon-Tyne, UK

P. Bettess

School of Engineering, University of Durham, UK and

A.G. Gibson

*Department of Mechanical, Materials and Manufacturing Engineering,
University of Newcastle-upon-Tyne, UK*

Received June 1995

Revised May 1996

Nomenclature

A	= pre-exponential factor, 1/sec.	N	= element shape function
c	= specific heat, J/kg K	\mathbf{N}	= shape function matrix
\mathbf{C}	= capacitance matrix	Q	= heat of decomposition, J/kg
E	= total number of elements	R	= gas constant, 8.314 kJ/kg-mole K
E_A	= activation energy, kJ/kg-mole		increasing rate of furnace
\mathbf{f}	= force (heat load) vector		temperature
h	= enthalpy, J/kg	t	= time, sec.
k	= thermal conductivity, W/m K	T	= temperature, °C
\mathbf{K}	= conductance matrix	\mathbf{T}	= temperature vector
l	= element length, m	V_f	= volume fraction
L	= laminate thickness, m	x	= spatial co-ordinate, m
m	= mass flux, kg/m ² -sec.	Ω	= element domain
n	= number of spatial nodes	Θ	= time step (weighting) parameter

The experimental work was funded by the MTD/Marinetech Research multi-sponsor programme on Cost-effective Use of Fibre Reinforced Composites Offshore, which is supported by the UK Marine Technology Directorate Ltd and a consortium of companies. Special thanks are due to the industrial sponsors of the programme for their support and for permission to publish some of the results presented here: Admiralty Research Establishment, AGIP(UK), Amerada Hess, Ameron bv, Amoco Research, Balmoral Group, Bow Valley Petroleum, BP Exploration, BP Research, Brasoil, British Gas, Ciba-Geigy, Conoco, Defence Research Agency, Dow Rheinmunster, Elf (Aquitaine), Elf (UK), Enichem (SPA), Exxon, Fibreforce Composites, Hunting Engineering Ltd, Kerr McGee Oil (UK) Ltd, MaTSU, Marine Technology Directorate Ltd, Mobil Research and Development, Mobil North Sea Ltd, Norsk Hydro, UK Ministry of Defence (Navy), UK Offshore Supplies Office, Phillips Petroleum, Shell Expro, Statoil, Total Oil Marine, UK Department of Energy, VSEL and Vosper Thornycroft.

The research programme involves the Universities of Glasgow, Liverpool, Salford, UMIST and Nottingham as well as Newcastle-upon-Tyne. Particular acknowledgement is also due to our collaborating partners, Professor J. M. Davies and his research group at Salford University and to OCF, Chomarar and DSM Resins for supply of materials and to Dr Y.-S. Wu for his valuable comments and related discussions.

The first author is also grateful to the Ministry of Culture and Higher Education of Iran for the financial support of his MSc studies.

International Journal of Numerical
Methods for Heat & Fluid Flow
Vol. 7 No. 6, 1997, pp. 609-625.
© MCB University Press, 0961-5539

ρ	= density, kg/m ³	T	= total
Δt	= time step size, sec.		= transpose
<i>Subscripts</i>		∞	= ambient
c	= conduction	<i>Superscripts</i>	
cr	= critical	e	= element
d	= density	i	= time position
f	= final	$-$	= average
	= furnace	\cdot	= partial derivative with respect to time
fr	= fibre reinforcement	<i>Abbreviations</i>	
g	= gas	FEM	= finite element method
j	= nodal position	GRP	= glass reinforced plastics
k	= Kelvin		
o	= initial values		
p	= constant pressure		
r	= resin matrix		

Note: The symbols defined above are subject to alteration on occasion

Introduction

Polymer composite materials are increasingly being used in offshore structures for forming fire and blast walls. These materials offer the possibility of significant savings in platform topside weight and in installation and maintenance costs. Additionally, they are resistant to the marine environment.

An attractive feature of the use of polymer composite materials in offshore structures is their fire-performance characteristics, and, therefore, the ability to predict the thermal response of these materials when exposed to fire is of great interest to researchers in this area. Furthermore, accurate knowledge of the fire performance of these materials is essential for the optimum design of thermal protection and evacuation systems in offshore structures.

Experiments in furnace tests and one-dimensional models of offshore panels and plates[1,2] have revealed that glass-fibre reinforced composites have excellent fire-resistance properties and give protection against fire for considerable periods. These properties are given by the decomposition of the resin material, which acts as a coolant. Among the various numerical methods suitable for thermal modelling, to date only the finite difference method[1,2] has been used to approximate the mathematical model of heat flow in these materials used in offshore components. However, because of widespread acceptance of the finite element method and its versatility for thermal problems involving non-linear governing equations and complex geometry and boundary conditions, this method has been chosen throughout this work.

Fire-resistance is generally defined as the time required for the average temperature of the cold face of a laminate to reach 160°C[3]. This value is, as might be expected, significantly shorter than the complete fire penetration time.

In many applications, the incident heat fluxes are large enough to cause both thermal decomposition of the resin material and surface recession so that these two processes have the same level of importance for an accurate prediction. Since the fire-resistance investigation of GRP panels needs a temperature rise at cold face of less than 160°C, the authors' interest, in this particular situation, lies in a mathematical model which includes only thermal decomposition of the resin

material and chemical reactions under 1,000°C with no surface recession occurring. The aim of this work is, therefore, to develop a general finite element technique to simulate the fire performance of GRP panels, relating to hydrocarbon fires, for use as blast and fire-resistant wall and floor elements of offshore modules. A finite element computer code, written by Bettess and Bettess[4], has been developed to apply the model to the standard thickness of a polyester-based GRP laminate, 10.9mm, used in offshore components. While the finite element model permits a rather general representation of the decomposition process and different boundary conditions, the specific solution presented here uses a first-order Arrhenius equation with constant thermal and transport properties throughout the decomposition process. Furthermore, the boundary condition at the hot (front) face of the laminate has been set up as an incident heat flux using an empirical formula[1] and the cold (back) face of the laminate has been insulated. The accuracy of the model has been evaluated by comparing the predicted temperature profiles with those obtained experimentally from furnace tests[1,2].

A model including a higher-order Arrhenius equation, temperature-dependent thermal and transport properties together with more accurate boundary conditions at the hot and cold faces and different configurations and materials of offshore components, will be presented in a future paper.

Problem description

The problem of predicting the behaviour of polymer composite materials exposed to a fire environment, particularly hydrocarbon fires in offshore structures, may be divided into two different parts: internal and external processes. The internal process includes all the physical and chemical processes which occur in any laminate of offshore components. The external process, however, addresses itself, first, to the determination of the shape, size and intensity of the flame in the boundary layer and, second, to heat transfer from this flame over the related laminate to others. Table I schematically depicts a cause-and-effect flow chart involving the internal process. According to the fire-resistance definition, the model developed in this work is intended to model the exposure time of the laminate hot face for which the laminate cold face remains at temperatures less than or equal to 160°C. Therefore, only the first five events of the internal process depicted in Table I have been considered.

Mathematical model

Several mathematical models have been developed which simulate the thermal response of materials undergoing decomposition and chemical reactions. The earliest models were proposed by Bamford and Malan[5], Murty and Blackshear[6], Murty[7], Kung[8] and Kansa *et al.*[9], to study chemical reactions and decomposition of wood in an attempt to develop fire-retardant timber. Later, the primary models were developed by a number of researchers[10-15] to account for the temperature-dependent thermal and transport properties, utilizing an n th order Arrhenius equation, decomposition due to carbon-silica reactions, accumulation of decomposition gases and the associated internal

HFF
7,6

612

Table I.
Flow chart listing the events in a laminate exposed to a fire environment

- (1) Radiation heat flux, caused by fire, into the front face of the laminate.
- (2) Pure transient heat conduction which is governed by the thermal properties of the laminate:
 - surface temperature is not sufficiently high, less than 200-300°C, dependent on material processing, composition and heating rate;
 - no chemical reactions occur;
 - temperature gradients versus time are initially sharp.
- (3) Degradation and chemical reactions (pyrolysis):
 - surface temperature is sufficiently high, 200-300°C but less than 1,000°C;
 - resin material degrades to form residual char and gaseous products;
 - the chemical reactions zone moves from heated surface through the material;
 - thermo-chemical expansion and/or contraction occur;
 - owing to the accumulation of the decomposition gases, internal pressurization of the material occurs;
 - the temperature gradients versus time reduce and the solid material is kept cool.
- (4) Final products and the completion of pyrolysis reactions:
 - temperature gradients versus time increase sharply.
- (5) Transient heat conduction which is governed by the thermal properties of the remaining glass reinforcement.
- (6) Carbon-silica reactions:
 - surface temperature exceeds 1,000°C;
 - the carbon residue from the pyrolysis reactions and the silica filler react chemically;
 - additional decomposition gaseous products are produced;
 - additional thermo-chemical expansion and/or contraction occur.
- (7) Surface recession:
 - the active material is eventually consumed at the heated surface;
 - the residue char reaches the melt or fail temperature.
- (8) Flame:
 - the remaining material burns out completely.

pressurization, thermo-chemical expansion and/or contraction, non-linear parameter estimation to determine certain thermal and transport properties, the absence of local-thermal equilibrium, and a coupled solution for the equation of energy, mass transport and momentum.

The model, which has been developed here to predict the behaviour of GRP laminates in a fire environment, is based on the mathematical model proposed by Henderson *et al.*[12]. In addition to the assumptions made in their model, further simplifying assumptions are applied as follows:

- (1) No intermediate compounds are produced when the resin material sublimes.
- (2) Thermal and transport properties of the laminate are constant during the decomposition process of the material, i.e. for the first four events indicated in Table I.
- (3) A first-order Arrhenius equation is adopted.

(4) A zero value for the final density of the resin material is applied.
A composite layer which is an infinite plate with finite thickness L is selected as a basis for the problem.

With the first and second assumptions, the phenomenon of energy conservation for one-dimensional heat transfer in the laminate of thickness L undergoing thermal decomposition is given by the following non-linear partial differential equation:

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - \dot{m}_r c_{pr} \frac{\partial T}{\partial x} - \frac{\partial \rho}{\partial t} (Q + h - h_r) \quad (1)$$

subject to the initial conditions

$$T(x, t) = T_\infty, \rho = \rho_o, \dot{m}_r = 0 \quad \text{for } 0 \leq x \leq L, t = 0 \quad (2)$$

and the following boundary conditions

$$T(x, t) = f(t) \quad \text{for } x = 0, t > 0 \quad (3a)$$

$$\frac{\partial T}{\partial x} = 0, \dot{m}_r = 0 \quad \text{for } x = L, t > 0 \quad (3b)$$

where $f(t)$ is the time-dependent temperature of the heated surface.

In this work the function $f(t)$ is defined on the basis of the empirical formula obtained from furnace tests by Wu *et al.*[1] as:

$$f(t) = \begin{cases} T_f(t) - 100R(t) & \text{if } t > 0 \\ 293 \text{ (K)} & \text{if } t = 0 \end{cases} \quad (3c)$$

where $T_f(t)$ is furnace temperature (K), and is expressed as:

$$T_f(t) = T_\infty + 273 + (1100 - T_\infty)R(t) \quad (3d)$$

and $R(t)$ is the increasing rate of furnace temperature and can be stated as:

$$R(t) = 1 - \exp \left[-\exp \left(0.71 \log \frac{t}{124.8} \right) \right] \quad (3e)$$

For a glass-fibre reinforced composite with aligned long or continuous reinforcement, the density, transverse thermal conductivity and specific heat are evaluated, by the law of mixtures, as:

$$\rho = V_f \rho_{fr} + (1 - V_f) \rho_r, \quad (4a)$$

$$\frac{1}{k} = \frac{V_f}{k_{fr}} + \frac{1 - V_f}{k_r} \quad (4b)$$

HFF
7,6

and:

$$\frac{1}{c_p} = \frac{V_f}{(c_p)_f} + \frac{1-V_f}{(c_p)_r} \quad (4c)$$

614

respectively.

Assuming no expansion of the laminate material and considering the second and third assumptions, the Arrhenius equation can be expressed as:

$$\frac{\partial \rho}{\partial t} = -A\rho \exp(-E_A/RT_k) \quad (5)$$

If the accumulation of the generated gas is ignored, the gas mass flux, m_g , at any spatial location and time, may be calculated as:

$$\dot{m}_g = \int_L^x (\partial \rho / \partial t) dx \quad (6)$$

The enthalpy of the laminate, h , and the generated gas enthalpy, h_g , have been assumed to be functions of temperature only and are given by:

$$h = \int_{T_\infty}^T c_p dT \quad (7a)$$

and:

$$h_g = \int_{T_\infty}^T c_{pR} dT \quad (7b)$$

respectively. According to the second assumption, these two equations can be simplified as:

$$h = c_p (T - T_\infty) \quad (8a)$$

$$h_g = c_{pR} (T - T_\infty) \quad (8b)$$

Equations (1), (5) and (6) form a set of non-linear partial differential equations which must be solved simultaneously using the initial and boundary conditions, Equations (2) and (3), for ρ , m_g and T respectively. Both density and its loss rate are calculated by the Arrhenius equation. The model is extremely sensitive to these values. This is to be expected since the rate of energy consumption or addition resulting from the decomposition is proportional to $\partial \rho / \partial t$.

Finite element formulation

The set of non-linear partial differential equations, equations (1), (5) and (6) developed in the previous section, has been solved numerically using a mixed explicit-implicit finite element technique[16]. With regard to this technique, the

density loss rate, density and gas mass flux are evaluated explicitly, whereas the nodal temperatures are computed using an implicit formulation.

Density loss rate, density and gas mass flux

In order to enter the density loss rate, density and gas mass flux equations into the finite element formulation, two notations have been used: time position, i , and nodal or space position, j .

Using the above notations and equations (5) and (6), the density loss rate and gas mass flux can be represented as follows:

$$\left(\frac{\partial \rho}{\partial t}\right)_j^{i+1} = \frac{\rho_j^{i+1} - \rho_j^i}{\Delta t} \tag{9}$$

$$(\dot{m}_g)_j^{i+1} = - \sum_{j=n_s}^{n_r} \left(\frac{\rho_j^{i+1} - \rho_j^i}{\Delta t}\right) \Delta x_j \tag{10}$$

where

$$\frac{\rho_j^{i+1} - \rho_j^i}{\Delta t} = -A \rho_j^i \exp\left[-E_A/R(T_k)_j^i\right] \tag{11}$$

Additionally, using equation (11), the density at each time and nodal position can be calculated as:

$$\rho_j^{i+1} = \rho_j^i \left\{1 - A \Delta t \exp\left[-E_A/R(T_k)_j^i\right]\right\} \tag{12}$$

Substitution of equations (9)-(12) into equation (1) results in:

$$\rho_j^{i+1} c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} - (\dot{m}_g)_j^{i+1} c_{pg} \frac{\partial T}{\partial x} - \left(\frac{\partial \rho}{\partial t}\right)_j^{i+1} (Q + h - h_g) \tag{13}$$

Element equations

The solution domain, which is a one-dimensional body of thickness L , is divided into E linear elements of the same length, l , and two nodes each.

The governing differential equation, equation (13), is cast into a matrix equation by applying the weighted residuals approach of the finite element method[17] and by approximating the temperature within each element as:

$$T(x,t) = \mathbf{N}(x)\mathbf{T}(t) \tag{14}$$

The weighted residuals approach requires that:

HFF
7,6

$$\int_{\Omega^e} \left[\rho_j^{i+1} c_p \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} + (\dot{m}_g)_j^{i+1} c_{pg} \frac{\partial T}{\partial x} + \left(\frac{\partial \rho}{\partial t} \right)_j^{i+1} (Q + h - h_g) \right] \mathbf{N} d\Omega^e = 0 \quad (15)$$

According to the definition of the solution domain, equation (15) can be rewritten as:

616

$$\int_0^l \left[\rho_j^{i+1} c_p \frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} + (\dot{m}_g)_j^{i+1} c_{pg} \frac{\partial T}{\partial x} + \left(\frac{\partial \rho}{\partial t} \right)_j^{i+1} (Q + h - h_g) \right] \mathbf{N} dx = 0 \quad (16)$$

We substitute equations (8a), (8b) and (14) into equation (16) and use integration by parts in one dimension, so that after some manipulation the resulting element equations become:

$$\begin{aligned} & \int_0^l (\mathbf{N}^T \rho_j^{i+1} c_p \mathbf{N}) dx \mathbf{T} + \int_0^l \left[\left(\frac{\partial \mathbf{N}}{\partial x} \right)^T k \left(\frac{\partial \mathbf{N}}{\partial x} \right) + \mathbf{N}^T (\dot{m}_g)_j^{i+1} c_{pg} \frac{\partial \mathbf{N}}{\partial x} \right. \\ & \left. + \mathbf{N}^T \left(\frac{\partial \rho}{\partial t} \right)_j^{i+1} (c_p - c_{pg}) \mathbf{N} \right] dx \mathbf{T} + \int_0^l \mathbf{N}^T \left(\frac{\partial \rho}{\partial t} \right)_j^{i+1} [Q - (c_p - c_{pg}) T_\infty] dx = \mathbf{0} \end{aligned}$$

or:

$$\mathbf{C} \mathbf{T} + (\mathbf{K}_c + \mathbf{K}_g + \mathbf{K}_d) \mathbf{T} + \mathbf{f} = \mathbf{0}$$

or:

$$\mathbf{C} \mathbf{T} + \mathbf{K} \mathbf{T} + \mathbf{f} = \mathbf{0} \quad (17)$$

where \mathbf{C} , \mathbf{K}_c , \mathbf{K}_g and \mathbf{K}_d can be written, in global co-ordinates, as:

$$\mathbf{C} = \int_0^l \mathbf{N}^T \rho_j^{i+1} c_p \mathbf{N} dx \quad (18a)$$

$$\mathbf{K}_c = \int_0^l \left(\frac{\partial \mathbf{N}}{\partial x} \right)^T k \left(\frac{\partial \mathbf{N}}{\partial x} \right) dx \quad (18b)$$

$$\mathbf{K}_g = \int_0^l \mathbf{N}^T (\dot{m}_g)_j^{i+1} c_{pg} \frac{\partial \mathbf{N}}{\partial x} dx \quad (18c)$$

$$\mathbf{K}_d = \int_0^l \mathbf{N}^T \left(\frac{\partial \rho}{\partial t} \right)_j^{i+1} (c_p - c_{pg}) \mathbf{N} dx \quad (18d)$$

$$\mathbf{f} = \int_0^l \mathbf{N}^T \left(\frac{\partial \rho}{\partial t} \right)^{i+1} \left[Q - (c_p - c_{pg}) T_\infty \right] dx$$

(18e)

The element matrices and the element vector, equations (18a)-(18e), are evaluated numerically using Gauss-Legendre quadrature.

It should be noted that the boundary condition at the laminate hot face is applied directly to the unknown temperature vector at each step of the computation. Furthermore, the insulated boundary condition at the laminate cold face adds only a zero value to the heat load vector.

617

Time step algorithm

A solution to the matrix equation, equation (17), has been given by Huebner *et al.*[18] and Zienkiewicz and Taylor[17] using the weighted residuals approximation. This solution is represented as:

$$(\mathbf{C} + \Delta t \Theta \mathbf{K}) \mathbf{T}_{n+1} = [\mathbf{C} - \Delta t (1 - \Theta) \mathbf{K}] \mathbf{T}_n - \Delta t \bar{\mathbf{f}}$$

(19)

where subscripts n and $n+1$ denote known and unknown quantities respectively. Furthermore, $\bar{\mathbf{f}}$ denotes an average value of \mathbf{f} given by:

$$\bar{\mathbf{f}} = \bar{\mathbf{f}}_n + \Theta (\bar{\mathbf{f}}_{n+1} - \bar{\mathbf{f}}_n)$$

(20)

It is clear that, in general, at each step of computation, a full system of equations needs to be solved. Thus, for an appropriate choice of Θ , the matrix equation, equation (19), can be solved using a non-symmetric profile matrix solver together with the active column method. A Crank-Nicolson solution with time step parameter, Θ , of value 0.5 has been used to solve the matrix equation implicitly and compute the nodal temperatures after each iteration.

Critical time step size

Zienkiewicz and Taylor[17] introduce the critical time step size, Δt_{cr} , for an explicit scheme of the finite element method as:

$$\Delta t_{cr} = \frac{2}{1 - 2\Theta} \frac{l^2 \rho c_p}{3k}$$

(21)

The overall computer code logic

The key operations flow chart of the finite element computer code developed in this work is shown in Table II.

Application

Material properties

The proposed mathematical and numerical models can be used to simulate the thermal response of a variety of decomposing polymer composite materials

HF
7,6

618

Table II.
Flow chart listing the steps for the heat transfer analysis section of the finite element computer code

-
- (1) Initialize the system matrices, vectors and principle variables.
 - (2) Read finite element mesh data, material properties, including transient heat conduction and decomposition data, and control ID value to perform different solutions.
 - (3) Start time-dependent calculation and determine furnace temperature according to equation (3).
 - (4) Calculate the density loss rate, density and gas mass flux regarding the control ID value for the required solution.
 - (5) Choose a suitable option according to the input element type and create element matrices, **K** and **C**, and element force vector, **f**, using equation (18):
 - get element shape functions and their derivatives;
 - calculate the element matrices and element force vector contributions regarding the input control ID value;
 - integrate the element matrices and element force vector and add the results to the previous values, then go to first sub-step of (5);
 - choose a suitable solution of the finite element method, i.e. explicit, implicit (backward difference) or the Crank-Nicolson.
 - (6) Assemble the system matrices and force vector, if any.
 - (7) Calculate and update time step size, Δt , using equation (21) or consider the chosen value for the implicit solutions.
 - (8) Form system equations according to equation (17).
 - (9) Calculate the hot face temperature and apply the boundary conditions.
 - (10) Solve the matrix equation using equations (19) and (20) and find temperatures and density losses.
 - (11) Add time step size, determined in step (7), then go to step (3), otherwise stop.
-

given the appropriate material properties. Further, recent indications have revealed that, among a wide range of these materials used in offshore components for fire protection and other applications, phenolic-based and polyester-based composites are superior to the others. These two polymer composites, in addition to their marine environment resistance characteristics, offer an attractive combination of mechanical strength and low cost in construction, installation and maintenance.

The material selected in this work is polyester-based GRP consisting of 0.45 and 0.55 per cent glass-fibre reinforcement and polyester resin respectively. This material has been chosen for the study because most of the material properties required as input to the numerical model have already been determined. However, a great deal of effort has been expended to assemble an accurate set of properties for polyester-based GRP. These properties were collected from a number of sources, mostly Wu *et al.*[1], and are summarized in Table III.

With regard to the second assumption made in the Mathematical model section, the material properties of the laminate are evaluated using equations (4a)-(4c) for the first four events indicated in Table I. At the stage of the fifth

Property	Value	Model for GRP fire-performance
Pre-exponential factor, A (1/sec)	1.0×10^3	619
Specific heat (glass-fibre), $(c_{\rho})_{fr}$ (J/kg K)	760.0	
Specific heat (polyester resin), $(c_{\rho})_r$ (J/kg K)	1,600.0	
Specific heat (gas), $c_{\rho g}$ (J/kg K)	2,386.5	
Activation energy, E_A (kJ/kg-mole)	0.5×10^5	
Thermal conductivity (glass-fibre, bulk), k_{fr} (W/m K)	1.04	
Thermal conductivity (polyester resin), k_r (W/m K)	0.20	
Heat of decomposition, Q (J/kg)	2.3446×10^5	
Gas constant, R (kJ/kg-mole K)	8.314	
Ambient temperature, T_{∞} ($^{\circ}$ C)	20.0	
Volume fraction, V_f	0.45	
Density (initial), ρ_o (kg/m ³)	1,812.0	
Density (glass fibre), ρ_{fr} (kg/m ³)	2,560.0	
Density (polyester resin), ρ_r (kg/m ³)	1,200.0	

Table III.
Material properties for polyester-based GRP

event, however, transient heat conduction is followed by the properties of the glass-fibre instead.

Finite element mesh data

In order to implement the finite element computer code for the solution to the one-dimensional standard polyester-base GRP laminate of thickness 10.9mm, linear two-node elements of the same length have been adopted. The finite element mesh data used in this work are presented in Table IV.

The critical time step size, Δt_{cr} , is calculated using equation (21). According to this equation and considering the changes of the laminate density governed by equation (12), the critical time step size decreases at each step of the computation. This value should, therefore, be updated after each step of computation whenever an explicit finite element solution is implemented. Using the initial value of the laminate density, ρ_o presented in Table III, 1,812.0 (kg/m³), the initial value of the

Type	Value	
Total number of elements, E	10	Table IV. Finite element mesh data
Total number of nodes, n_T	11	
Total number of degrees of freedom	11	
Laminate thickness, L (m)	1.09×10^{-2}	
Element length, l (m)	1.09×10^{-3}	

critical time step size would be 4.46 seconds. For a linear governing equation, an implicit method such as the Crank-Nicolson solution is unconditionally stable for any value of the time step size. The non-linear case considered in this work, however, requires iteration at each time interval. If the time step size chosen is too large, oscillations and then instabilities will result. Through a trial-and-error procedure, it was determined that time step size of 5.0 seconds could be used for the Crank-Nicolson solution with no instability occurring.

Results and discussion

In Figure 1, the predicted temperatures are plotted as a function of time at four spatial locations along with the experimental data, obtained from furnace tests by Wu *et al.*[1,2]. Figure 1 shows a close agreement between the predicted temperatures and the experimental data. The average temperature difference for all the data points shown in Figure 1 is 29.6°C.

To show how each term of the element equation, equation (13), contributes in reducing the temperature rises through the laminate, four equations, i.e. pure transient heat conduction, transient heat conduction and gas mass flux, transient heat conduction and density loss, and full decomposition process have been considered. Figure 2 depicts the predicted temperatures of the laminate cold face versus time along with the experimental data using the above equations.

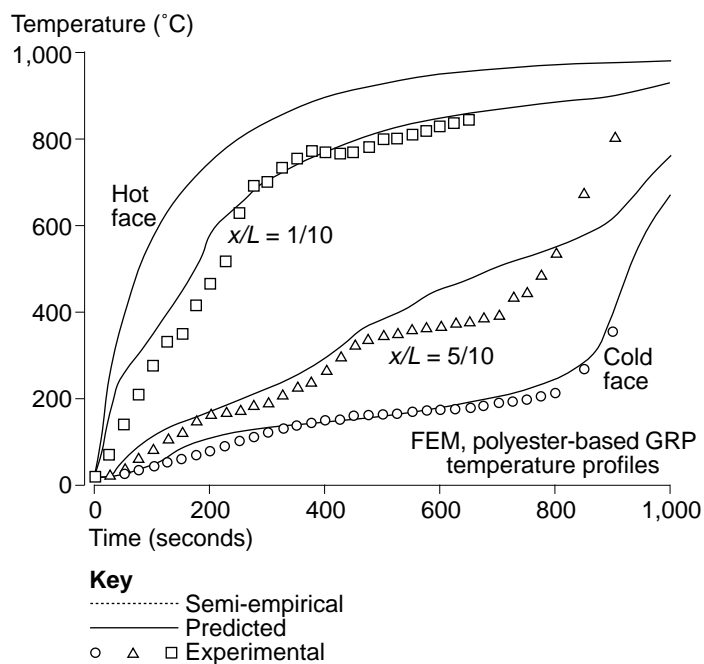


Figure 1. Comparison of the predicted and experimental temperature profiles using the Crank-Nicolson solution

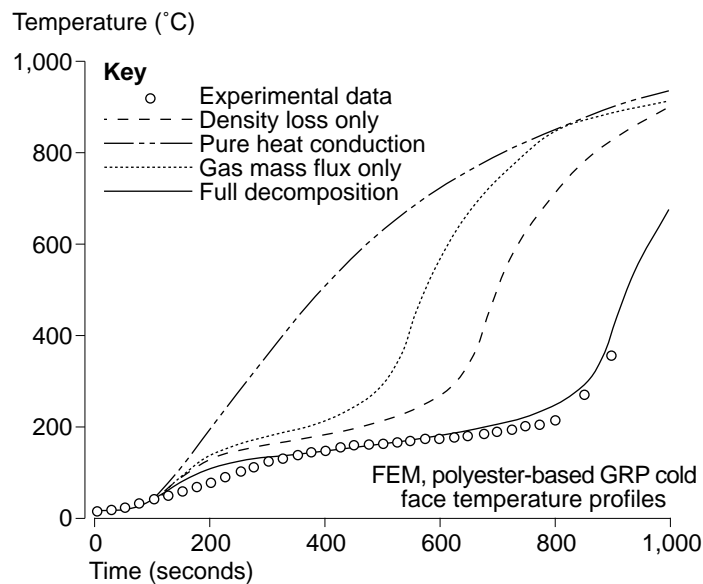


Figure 2. Comparison of the effects of different terms in the element equations on the predicted cold face temperatures together with experimental data

Considering the cold face temperature profile, Figure 1, it is possible to evaluate the fire-resistance behaviour of the laminate exposed to fire. According to the concept of the fire-resistance defined in the Introduction and also the results shown in Figure 1, the fire-resistance values of the Crank-Nicolson solution and experimental data are found as 474.77 and 449.50 seconds respectively. The accuracy of the numerical model can be verified by a very small difference of 25.27 seconds or 5.62 per cent between the predicted and experimental fire-resistance results.

In Figure 3, the predicted temperatures are plotted versus depth from the hot face to the cold face of the laminate at 100, 200, 400, 700 and 1,000 seconds.

In Figure 4, the predicted resin density and the laminate density fractions are plotted versus time from the hot face to the cold face of the laminate at four spatial locations. According to this figure, the completion of the pyrolysis reactions occurs at $\rho/\rho_0 = 0.64$. It is clear that for times greater than 125 seconds the pyrolysis reactions reach completion at the hot face of the laminate.

Finally, Figure 5 illustrates the predicted density loss rate, on the basis of the Arrhenius equation, versus time at four spatial locations. This figure clearly demonstrates the time derivatives of the density profiles presented in Figure 4. The feature presented by these density loss rate profiles in conjunction with the corresponding temperature and density profiles, Figures 3 and 4, provides some useful patterns followed by the pyrolysis reactions. The laminate layers progressively lying deeper from the hot face illustrate a progressively lower maximum pyrolysis rate and a progressively wider range of temperature in

HFF
7,6

622

Figure 3.
Location-dependent
temperature profiles

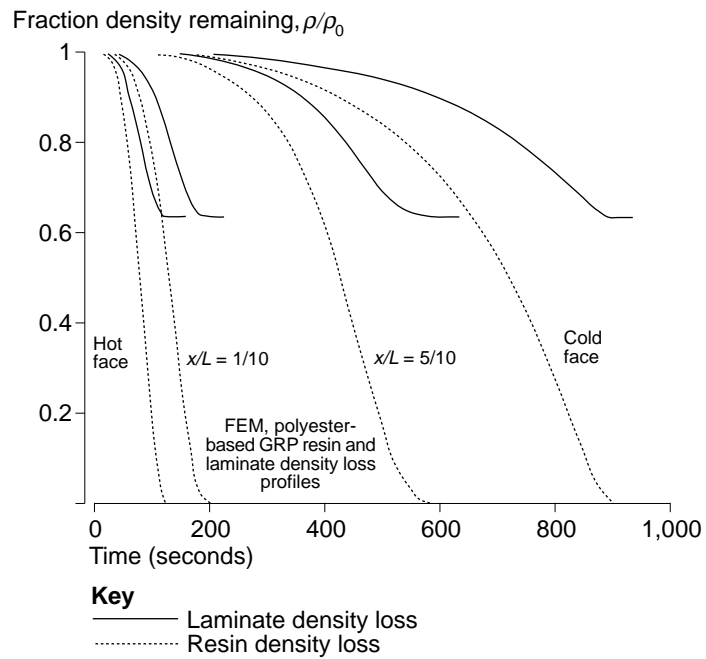
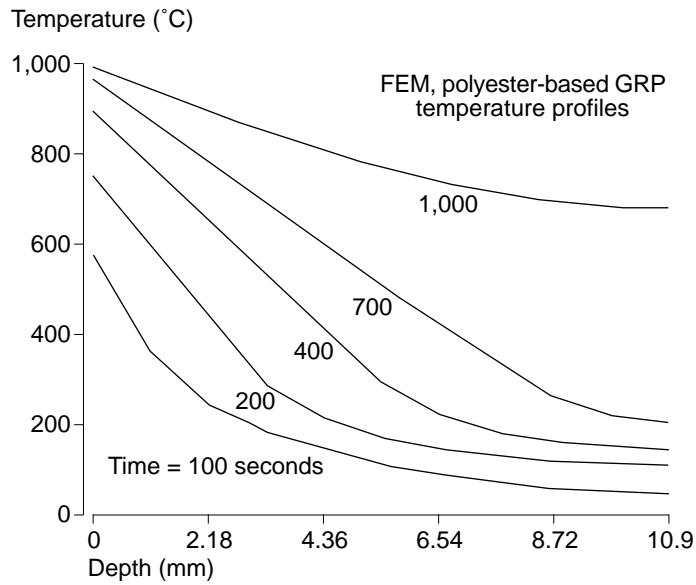


Figure 4.
Laminate and resin
density losses profiles

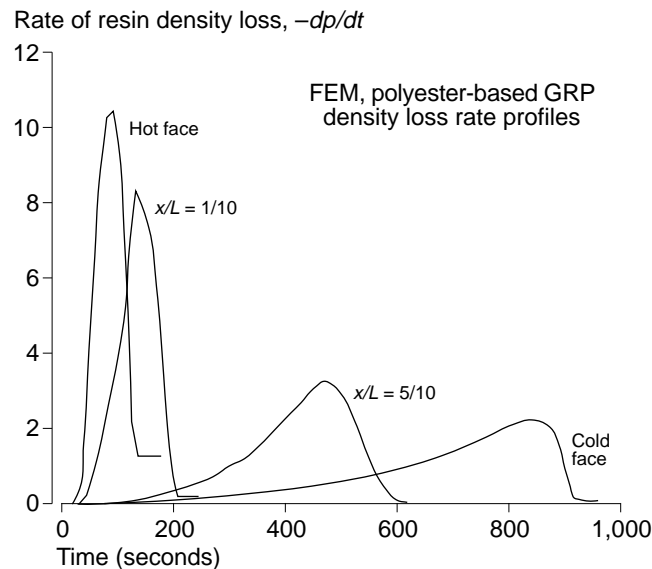


Figure 5.
Resin density loss rate
profiles

which pyrolysis reactions occur. In fact, the ratio of the maximum pyrolysis rate to the temperature at which it occurs is roughly constant. While the rate curves near the hot face are sharp, those at the intermediate spatial locations and near the cold face show rather smooth and regular changes with a constant rate for a certain duration.

Conclusions

Modelling the fire-performance of glass-fibre reinforced composites used in offshore components has two major benefits. First, it speeds up the design process and, thereby, reduces the cost of using this class of materials for fire-resistance purposes. Second, it improves the knowledge of how these materials behave when exposed to an unwanted fire.

The objective of this work was to investigate the applicability of the finite element method to analyse the thermal behaviour of polymer composite materials when exposed to fire. Basic concepts of the phenomena occurring in the material exposed to fire together with the mathematical model and finite element formulation were described in detail.

The finite element model associated with the computer code, developed during this study, was used to quantify the fire-performance and thermal response of a standard GRP laminate.

The predicted temperatures agreed with the experimental data with an average difference of 29.66°C. The accuracy of the model was also verified by comparing the predicted and experimental fire-resistance results. The

comparison reveals an extremely small difference of 25.27 seconds or 5.62 per cent. This model is, therefore, able to simulate fire performance characteristics for a large range of typical offshore components including different constructions and materials. It will enable accurate predictions of the lifetime of these components in severe offshore hydrocarbon fires. Finally, by using the finite element method, a wide range of geometries of composite components can easily be studied with little change to the computer code. Work on a more accurate one-dimensional model and also simple two- and three-dimensional models is continuing.

References

1. Wu, Y.-S., Wilcox, J.A.D., Gibson, A.G. and Bettess, P., *Design of Composite Panels for Mechanical and Fire Performance*, Final report prepared for British Gas plc, University of Newcastle-upon-Tyne, 1993.
2. Gibson, A.G., Wu, Y.-S., Chandler, H.W., Wilcox, J.A.D. and Bettess, P.A., "Model for the thermal performance of thick composite laminates in hydrocarbon fires", *Revue De L'Institut Français Du Pétrole*, Vol. 50 No. 1, 1995, pp. 69-74.
3. Sheno, R.A. and Wellicome, J.F., *Composite Materials in Maritime Structures, Practical Consideration*, Vol. 2, Cambridge University Press, Cambridge, 1993.
4. Bettess, P. and Bettess, J.A., *Finite Element Program*, University of Newcastle-upon-Tyne, 1986.
5. Bamford, C.H. and Malan, D.H., "The combustion of wood, Part 1", *Cambridge Phil. Soc. Proc.*, Vol. 42, 1946, pp. 166-82.
6. Murty, K.A. and Blackshear, P.L., "Pyrolysis effects in the transfer of heat and mass in thermally decomposing organic solids", *Eleventh Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, PA, 1967, p. 517.
7. Murty, K.A., "Thermal decomposition kinetics of wood pyrolysis", *Combustion and Flame*, Vol. 18, 1972, pp. 75-83.
8. Kung, H.C., "A mathematical model of wood pyrolysis", *Combustion and Flame*, Vol. 18, 1972, pp. 185-95.
9. Kansa, E.J., Perlee, H.E. and Chaiken, R.F., "Mathematical model of wood pyrolysis including internal forced convection", *Combustion and Flame*, Vol. 29, 1977, pp. 311-24.
10. Fredlund, B., "Modelling of heat and mass transfer in wood structures during fire", *Fire Safety Journal*, Vol. 20, 1993, pp. 39-69.
11. Henderson, J.B. and Wiecek, T.E., "A mathematical model to predict the thermal response of decomposing, expanding polymer composites", *J. of Comp. Mater.*, Vol. 21, 1987, pp. 373-93.
12. Henderson, J.B., Wiebelt, J.A. and Tant, M.R., "A model for the thermal response of polymer composite materials with experimental verification", *J. of Comp. Mater.*, Vol. 19, 1985, pp. 579-95.
13. Henderson, J.B., "An analytical and experimental study of the pyrolysis of composite ablative materials", PhD Thesis, Oklahoma State University, Stillwater, OK, 1980.
14. Florio Jr J., Henderson, J.B., Test, F.L. and Hariharan, R., "A study of the effects of the assumption of local-thermal equilibrium on the overall thermally-induced response of

decomposing, glass-filled polymer composite”, *Int. J. of Heat and Mass Transfer*, Vol. 34 No. 1, 1991, pp. 135-47.

15. Sullivan, R.M., “A coupled solution method for predicting the thermostructural response of decomposing, expanding polymeric composites”, *J. of Comp. Mater.*, Vol. 27 No. 4, 1993, pp. 408-34.
16. Looyeh, M.R.E., “Finite element simulation to one-dimensional heat flow in composite fire walls for offshore structures”, unpublished MSc dissertation, University of Newcastle-upon-Tyne, Newcastle-upon-Tyne, 1994.
17. Zienkiewicz, O.C. and Taylor, R.L., *The Finite Element Method: Solid and Fluid Mechanics, Dynamics and Non-linearity*, 4th ed., Vol. 2, McGraw-Hill, New York, NY, 1994.
18. Huebner, K.H., Thornton, E.A. and Byrom, T.G., *The Finite Element Method for Engineers*, 3rd ed., John Wiley & Sons, New York, NY, 1995.

Model for GRP
fire-performance